Gravitational Wave is Coming to Earth !

Group 6: Tang Hin Fung
Chan Matthew

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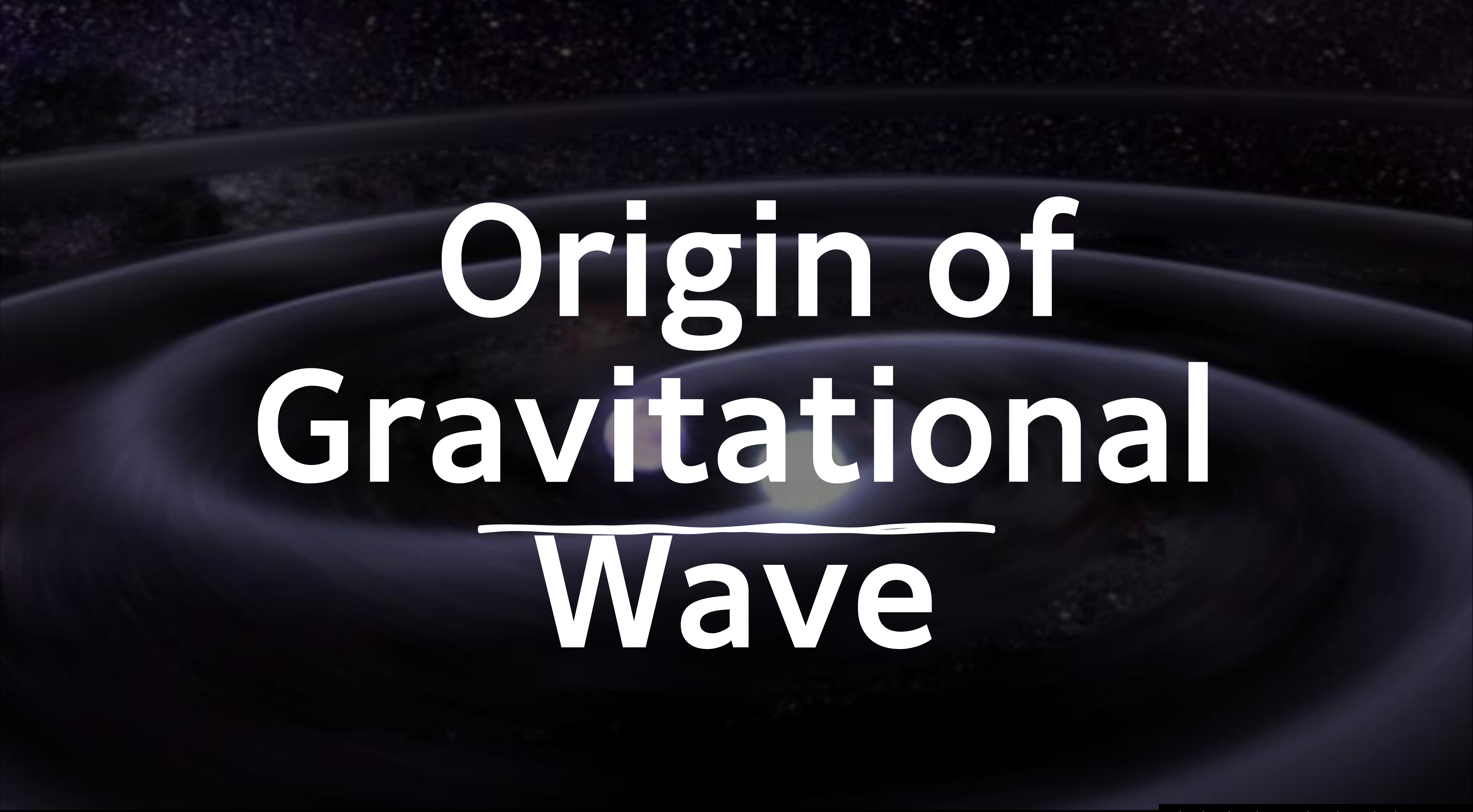
Origin of Gravitational Wave



Background History

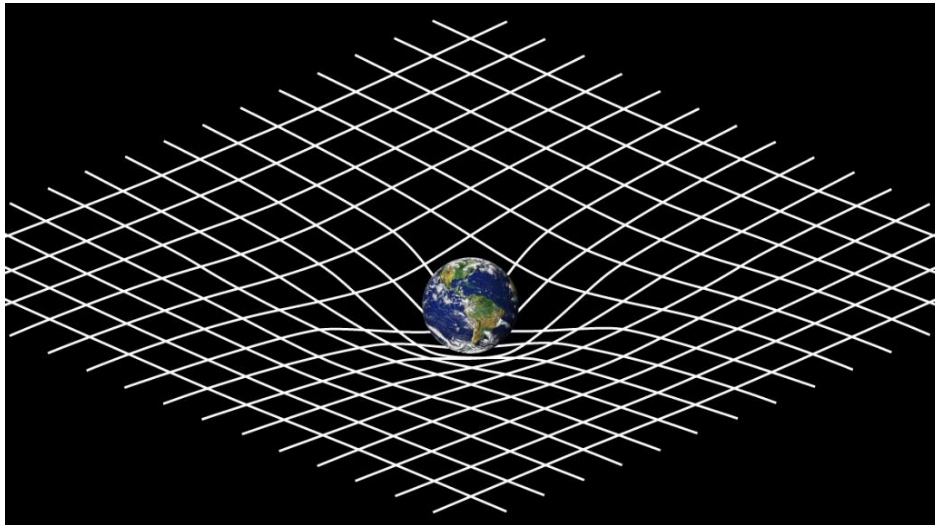
- In 1915 Einstein proposed "General Relativity (GR)"
  - -Describe gravity as curvature of spacetime

Einstein field equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

Einstein Tensor:
Describe spacetime curvature

Stress-Energy Tensor:
Describe the density, flux of energy and momentum.



Based on General Relativity, Einstein predicted Gravitational wave(GW) exist but…

In 1936, Einstein and Nathan Rosen submitted a paper to "*Physical review*" which claimed gravitational wave could not exist, but it was rejected.

Question:

How does General Relativity predict Gravitational waves?

Main idea:

Add a small perturbation to flat spacetime.

Linearized Gravity solutions

Starting Point.....

General Spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

A small perturbation of flat time

(Minkowski)Flat spacetime , no gravity involve

Physics behind:
Assuming that the waves produced by the sources are so weak that spacetime can be written as a small perturbation of flat spacetime.

Next step…

Connection coefficient $\Gamma^{\sigma}_{\mu\nu}$

Riemann curvature tensor $R^{\rho}_{\sigma\mu\nu}$

Ricci tensor $R_{\sigma\nu}$

After linearized gravity $\longrightarrow$?

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

Ricci scalar R

Einstein Tensor $G_{\mu\nu}$

Connection coefficient

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\alpha}(\partial_\nu g_{\alpha\mu} + \partial_\mu g_{\alpha\nu} - \partial_\alpha g_{\mu\nu})$$

$$\Big\downarrow \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} \eta^{\sigma\alpha}(\partial_\nu h_{\alpha\mu} + \partial_\mu h_{\alpha\nu} - \partial_\alpha h_{\mu\nu})$$

Riemann curvature tensor

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu(\Gamma^{\rho}_{\nu\sigma}) - \partial_\nu(\Gamma^{\rho}_{\mu\sigma}) + \Gamma^{\gamma}_{\nu\sigma}\Gamma^{\rho}_{\mu\gamma} - \Gamma^{\delta}_{\mu\sigma}\Gamma^{\rho}_{\nu\delta}$$

$$\Big\downarrow$$

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2}\eta^{\sigma\alpha}(\partial_\nu h_{\alpha\mu} + \partial_\mu h_{\alpha\nu} - \partial_\alpha h_{\mu\nu})$$

$$R^{\rho}_{\sigma\mu\nu} = \frac{1}{2}\eta^{\rho\alpha}(\partial_\mu\partial_\rho h_{\alpha\nu} - \partial_\mu\partial_\alpha h_{\nu\sigma} - \partial_\nu\partial_\sigma h_{\alpha\mu} + \partial_\nu\partial_\alpha h_{\mu\sigma})$$

Ricci tensor

$$R^{\rho}_{\sigma\mu\nu} = \frac{1}{2}\eta^{\rho\alpha}(\partial_\mu\partial_\rho h_{\alpha\nu} - \partial_\mu\partial_\alpha h_{\nu\sigma} - \partial_\nu\partial_\sigma h_{\alpha\mu} + \partial_\nu\partial_\alpha h_{\mu\sigma})$$

$$\Downarrow$$

$$\Box \equiv \eta^{\mu\alpha}\partial_\mu\partial_\alpha$$
$$h \equiv h^{\mu}_{\mu}$$

$$R_{\sigma\nu} = R^{\mu}_{\sigma\mu\nu} = \frac{1}{2}(\partial_\mu\partial_\sigma h^{\mu}_{\nu} + \partial_\nu\partial_\alpha h^{\alpha}_{\sigma} - \Box h_{\nu\sigma} - \partial_\nu\partial_\sigma h)$$

Ricci scalar

$$\color{red}{R_{\sigma\nu}} = \frac{1}{2}(\partial_\mu \partial_\sigma h^\mu_\nu + \partial_\nu \partial_\alpha h^\alpha_\sigma - \Box h_{\nu\sigma} - \partial_\nu \partial_\sigma h)$$

$$\Downarrow$$

$$\Box \equiv \eta^{\mu\alpha}\partial_\mu \partial_\alpha$$

$$h \equiv h^\mu_\mu$$

$$\color{red}{R} = \color{red}{R^\nu{}_\nu} = \partial_\mu \partial_\sigma h^{\mu\sigma} - \Box h$$

Einstein Tensor

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$R_{\sigma\nu} = R^{\mu}_{\sigma\mu\nu} = \frac{1}{2}(\partial_\mu \partial_\sigma h^{\mu}_{\nu} + \partial_\nu \partial_\alpha h^{\alpha}_{\sigma} - \Box h_{\nu\sigma} - \partial_\nu \partial_\sigma h)$$

$$R = R^{\nu}{}_{\nu} = \partial_\mu \partial_\sigma h^{\mu\sigma} - \Box h$$

$$h_{\alpha\beta} \equiv \bar{h}_{\alpha\beta} + \frac{1}{2} \eta_{\alpha\beta} h$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$G_{\mu\nu} \equiv \frac{1}{2}(\partial^{\mu} \partial_\alpha \bar{h}_{\alpha\nu} + \partial^{\alpha} \partial_\nu \bar{h}_{\mu\alpha} - \partial^{\alpha} \partial_\alpha \bar{h}_{\mu\nu} - \eta_{\mu\nu} \partial^{\alpha} \partial^{\beta} \bar{h}_{\alpha\beta})$$

Lorenz Gauge transformation

$$G_{\mu\nu} \equiv \frac{1}{2}(\partial^{\mu}\partial_{\alpha}\bar{h}_{\alpha\nu} + \partial^{\alpha}\partial_{\nu}\bar{h}_{\mu\alpha} - \partial^{\alpha}\partial_{\alpha}\bar{h}_{\mu\nu} - \eta_{\mu\nu}\partial^{\alpha}\partial^{\beta}\bar{h}_{\alpha\beta})$$

Lorenz Gauge condition:

$$\partial_{\beta}\bar{h}_{\alpha\beta} = 0$$

$$G_{\mu\nu} \equiv \frac{-1}{2}\eta_{\mu\nu}\partial^{\alpha}\partial^{\beta}\bar{h}_{\alpha\beta}$$

Gravitational Wave is indeed wave

Einstein field equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

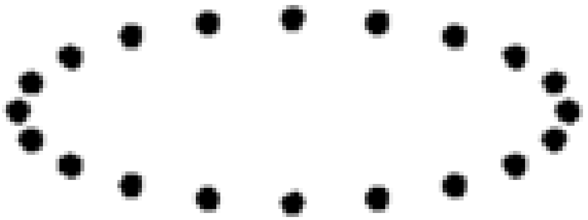
$T_{\mu\nu}=0$ (For flat spacetime)

$$G_{\mu\nu} \equiv \frac{-1}{2}\eta_{\mu\nu}\partial^{\alpha}\partial^{\beta}\bar{h}_{\alpha\beta}$$

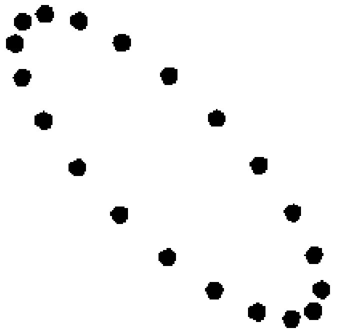
$$\eta_{\mu\nu}\partial^{\alpha}\partial^{\beta}\bar{h}_{\alpha\beta} = 0$$

$$\nabla^2\bar{h}_{\alpha\beta} = \frac{1}{c^2}\frac{\partial^2\bar{h}_{\alpha\beta}}{\partial t^2}$$ (Wave equation)

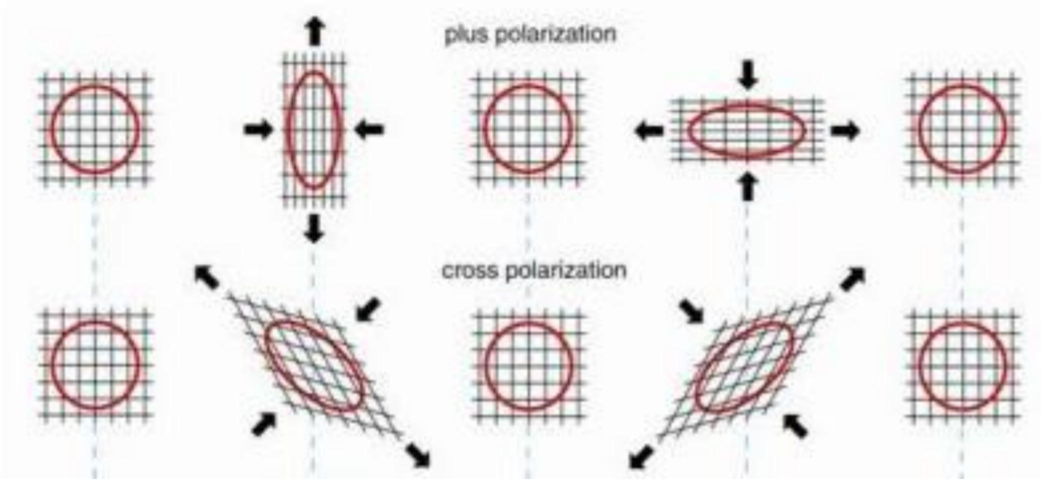
Polarization of gravitational wave: Plus and cross polarization



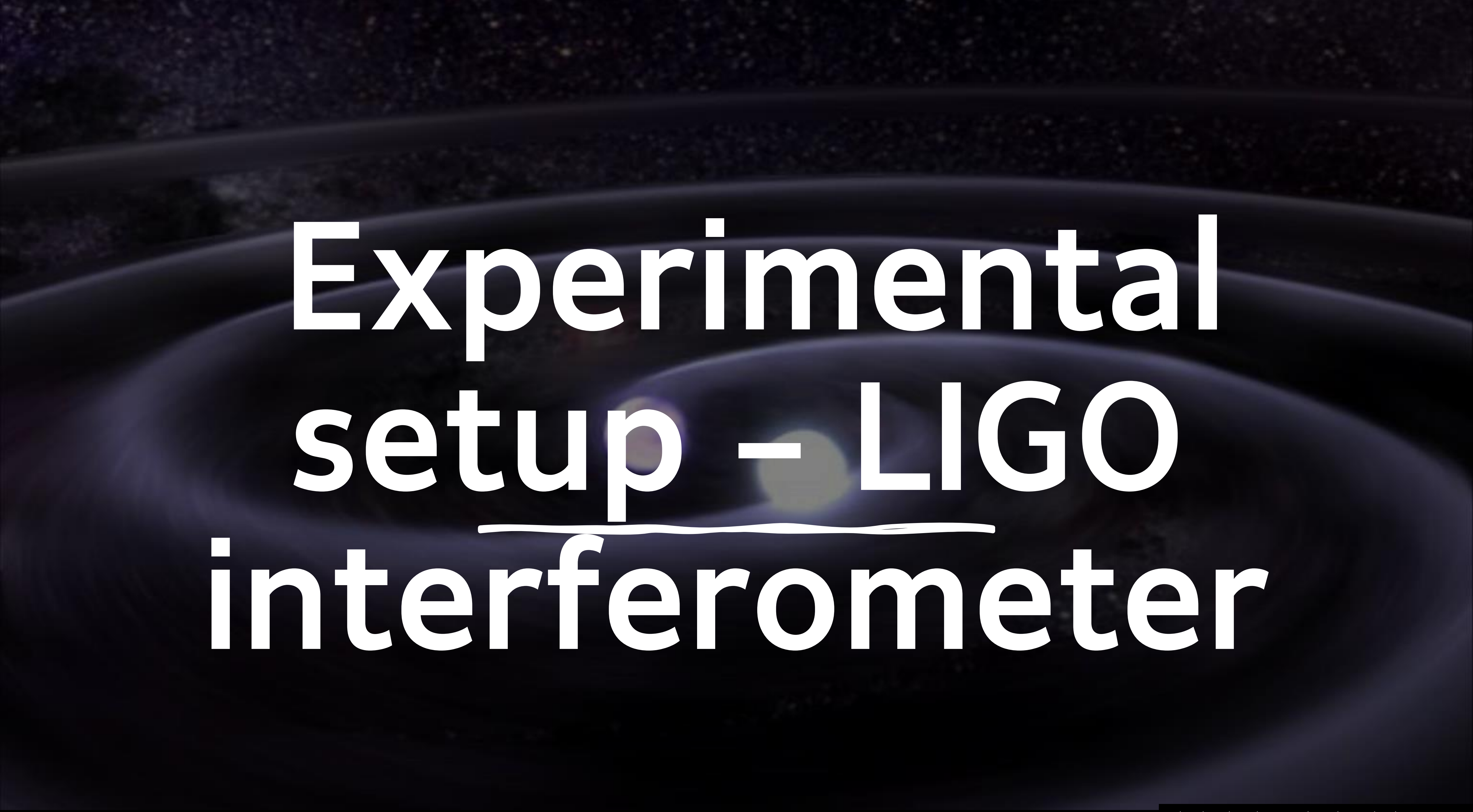
Plus polarization



Cross polarization



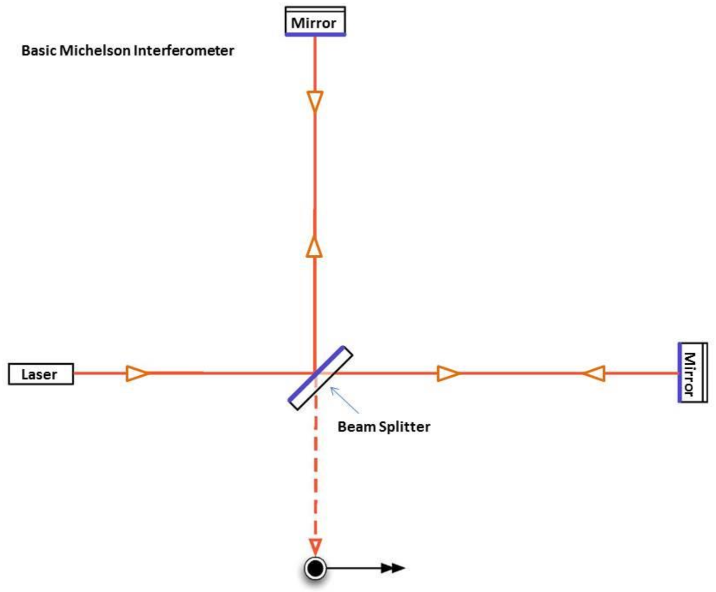
Gravitational Wave would stretch and compress spacetime in different direction.



Experimental setup – LIGO interferometer

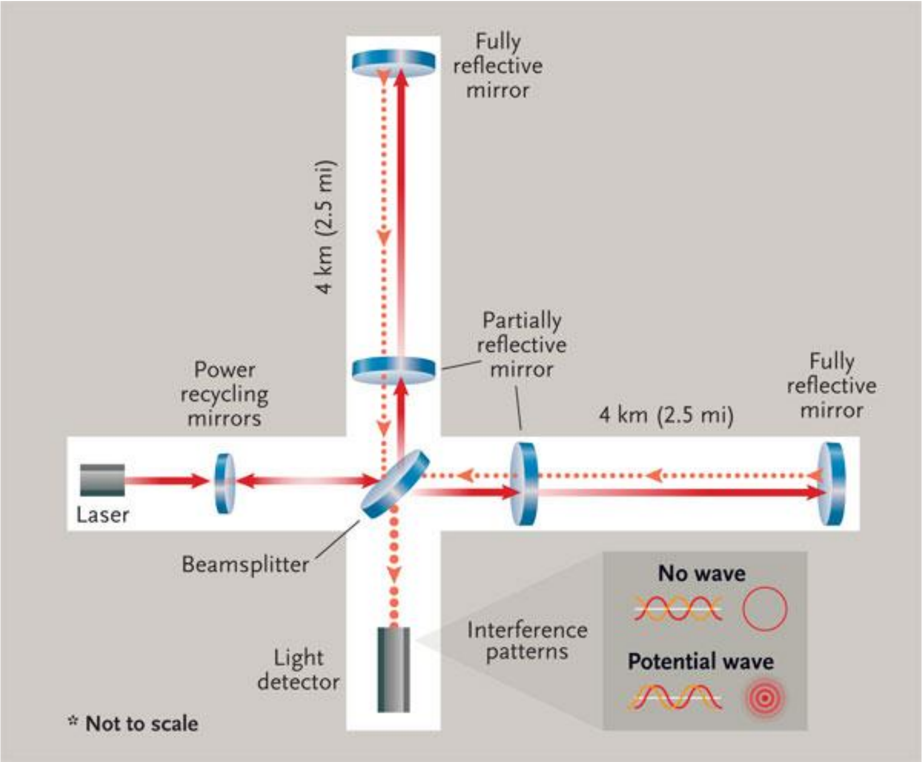
What is Interferometer?

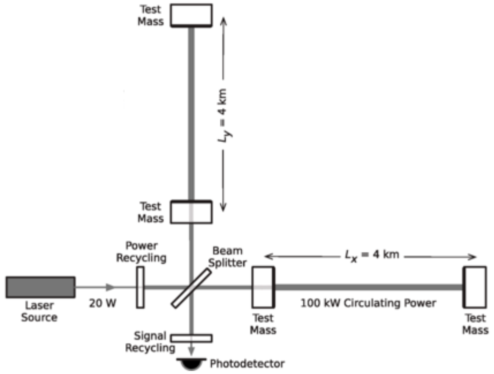
- Interference + meter
- Michelson Interferometer
  - Splitting beams into 2 arms
  - Path difference
  - Two beams recombined
  - Forming constructive or destructive interference



How is GW detected?

- No GW -> destructive interference
- GW -> stretch the arm length
  - Amplitude of GW is reflected by path difference
  - Start to have constructive interference
  - Intensity increase

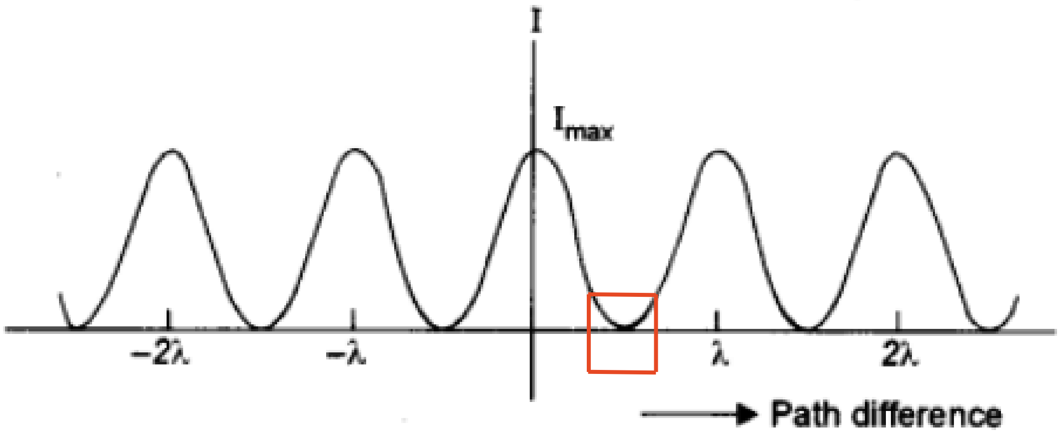




How Small is GW Amplitude?

- Strain of GW h: $\sim 10^{-21}$

- $\Delta L = Lh = n\lambda$
- $\lambda \approx 10^{-6}, L \approx 10^{3}$
- $n \ll 1$



- Didn’t finish a complete constructive interference

Signal to Noise

- Signal to noise ratio (SNR)

$$SNR \propto \frac{L\sqrt{P_{arm}}}{\lambda} h$$

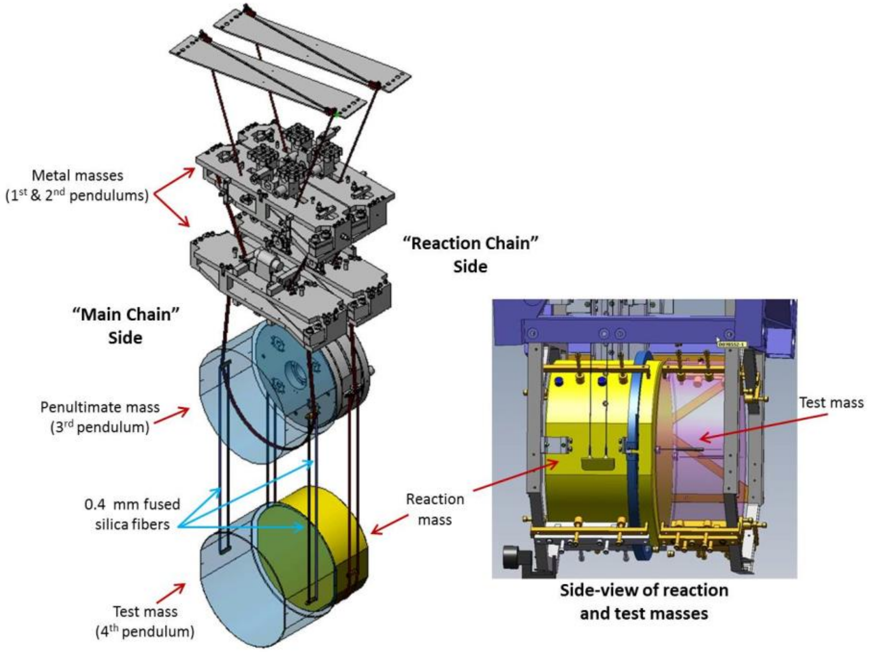
- L:cavity length
- P_{arm}: laser power in the arms
- λ: laser wavelength
- h: GW amplitude

How LIGO Increases Sensitivity?

- 4km cavity length
  - light reflected for about 300 times
  - Indirectly increase arm length/path difference
- High laser power
  - Power recycling mirror: reflect light from interferometer back to the interferometer
  - increase sensitivity/resolving power
- Wavelength of laser light
  - Stable
  - Optical coating
  - Detector bandwidth and noise

How LIGO Suppresses Noise?

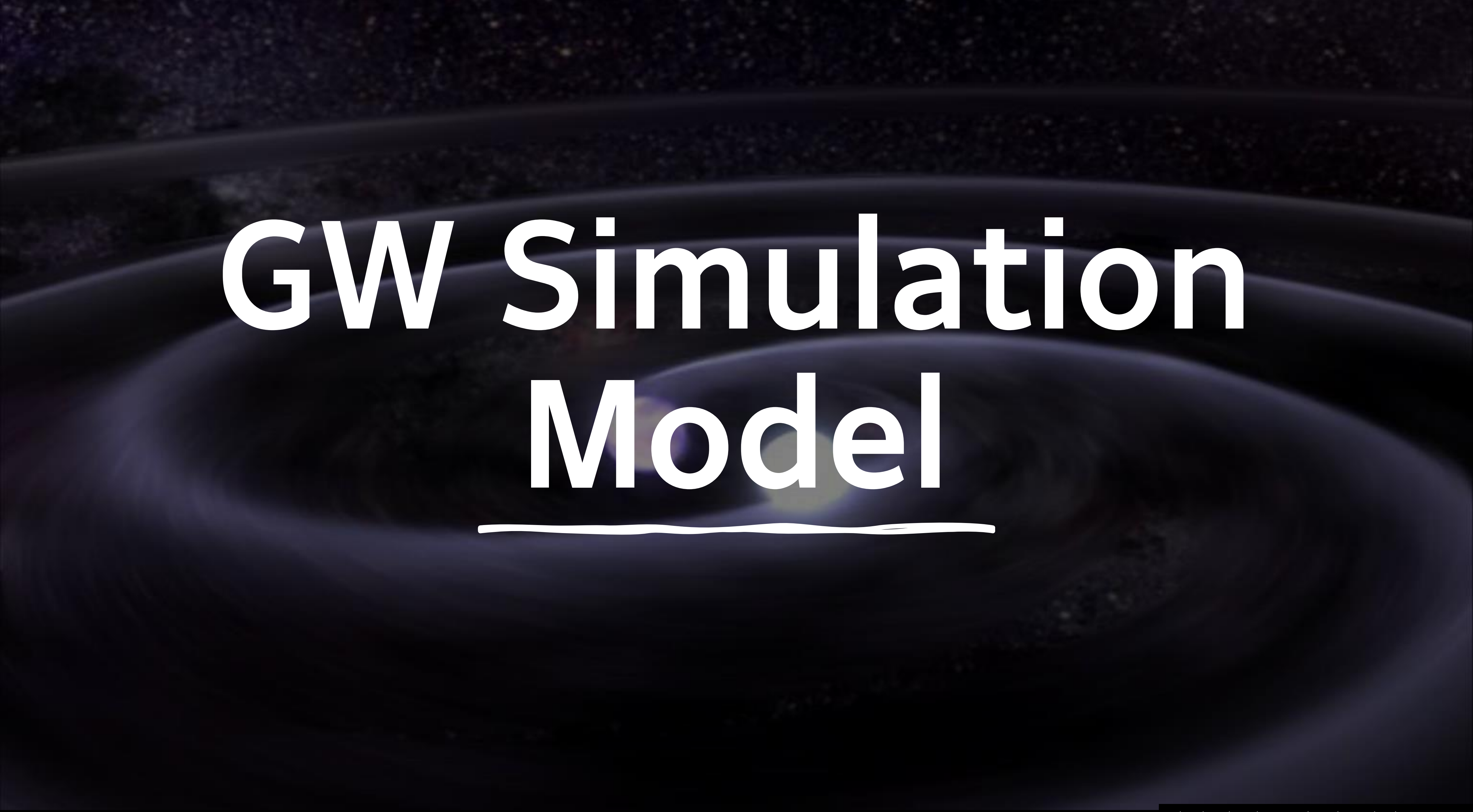
- Ultra-High Vacuum (passive isolation)
  - Vibration of sound waves
  - Thermal isolation
- Optical Suspension (passive isolation)
  - quadruple-pendulum system
  - Large masses
- Control System (active isolation)
  - vibration-sensors sense frequencies of environmental vibrations
  - Counteracts the vibration



LIGO on Earth

- Control Experiment

- Hanford
- Livingston



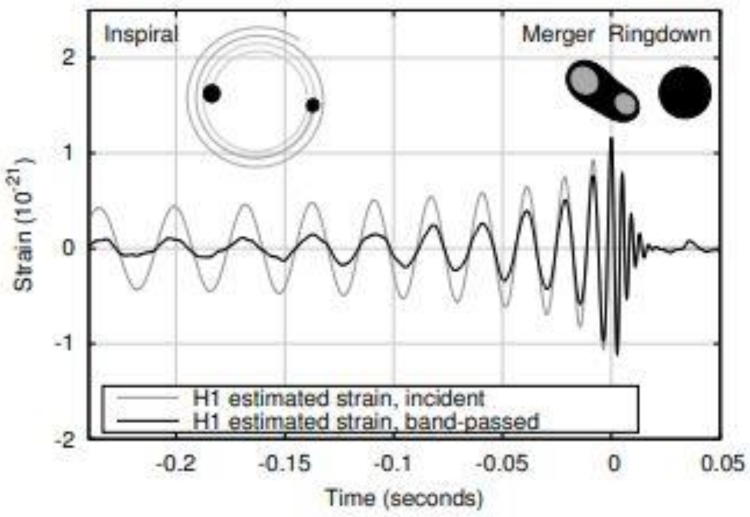
GW Simulation Model

Why We Need Simulation?

- Signal and Noise are mixed
- Target the signal frequency in experimental setup
- Use theoretical model to filtrate the signal

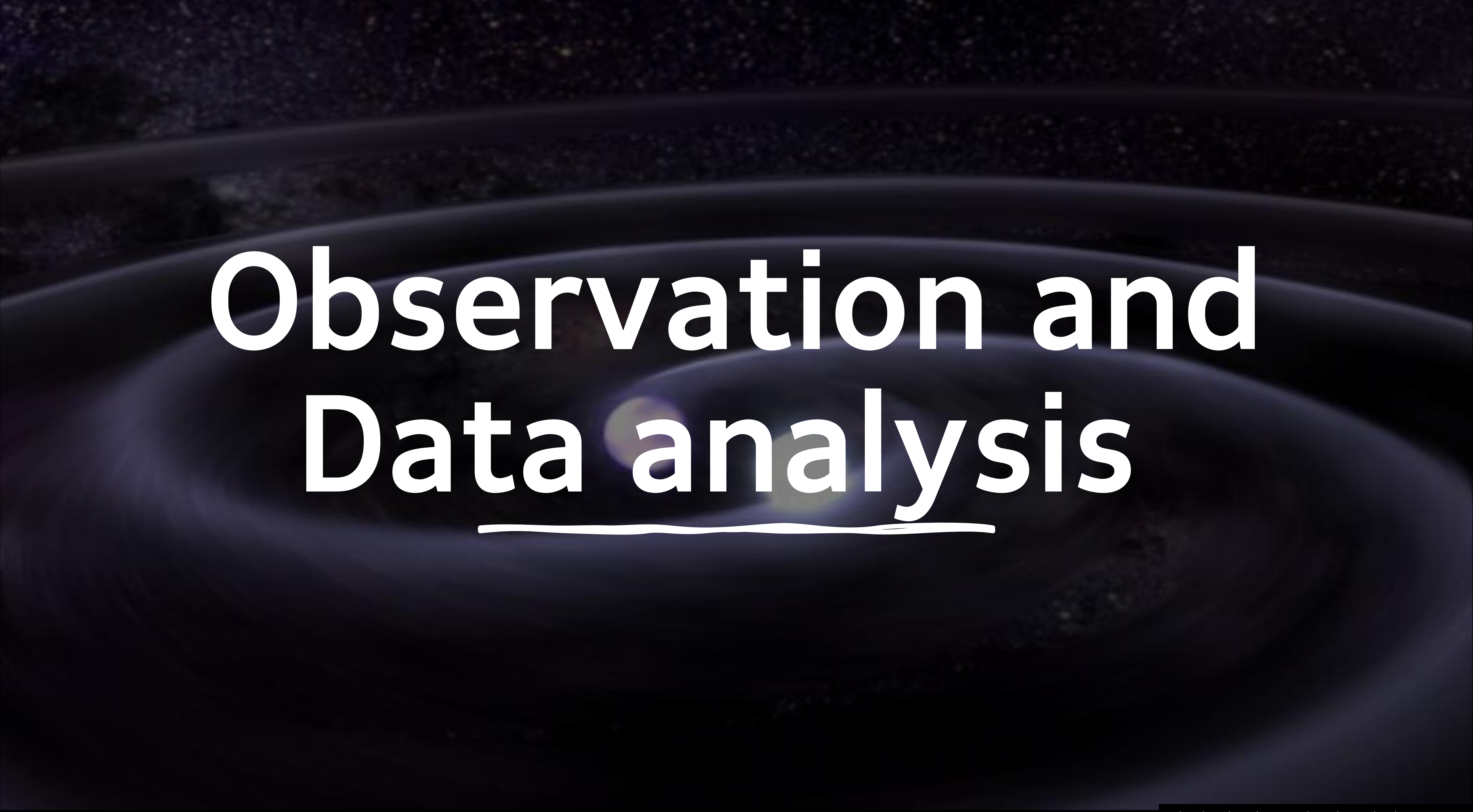
Simulation Method

- Post Newtonian Approximation
  - Weak regime
  - Expand metric tensor in small parameters
- Numerical Relativity
  - Strong regime
  - Solving the Einstein Field Equation (PDE) by numerical method
  - 15 parameters:
    - Mass*2
    - Spin*6
    - Source position and reference time for the remaining



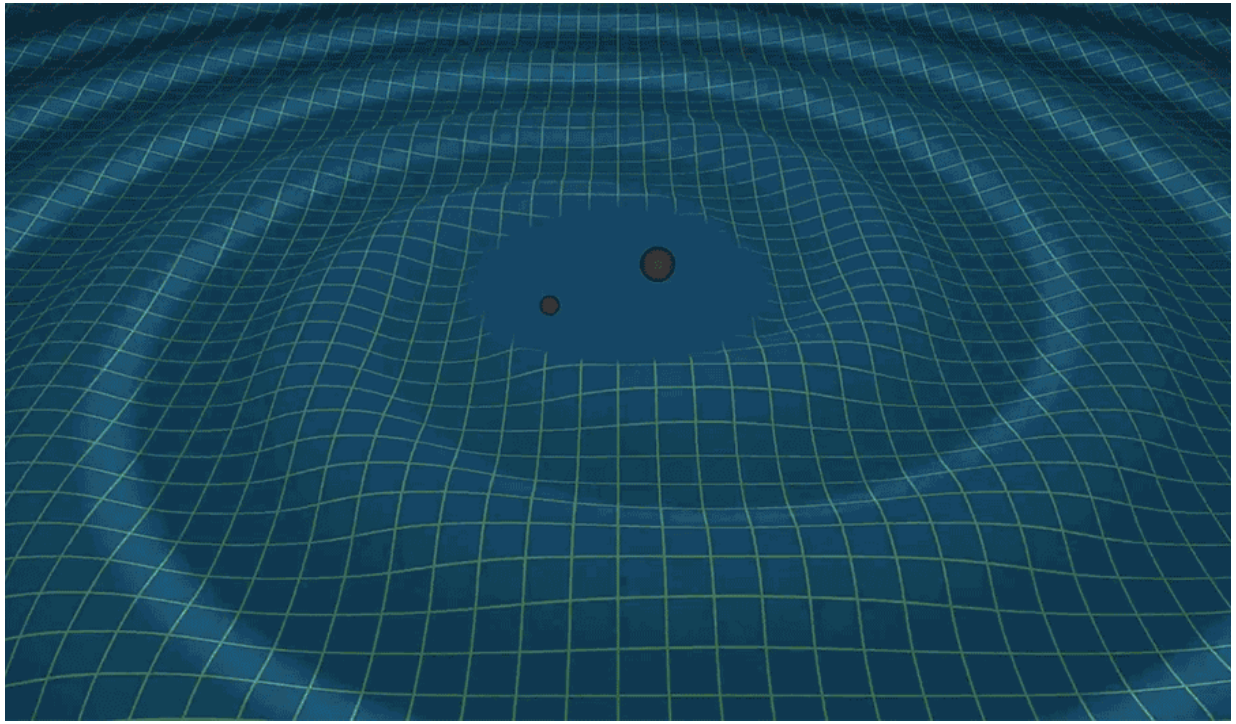
Directly comparing GW150914 with numerical solutions of Einstein's equations for binary black hole coalescence

Observation and Data analysis



First detection of GW

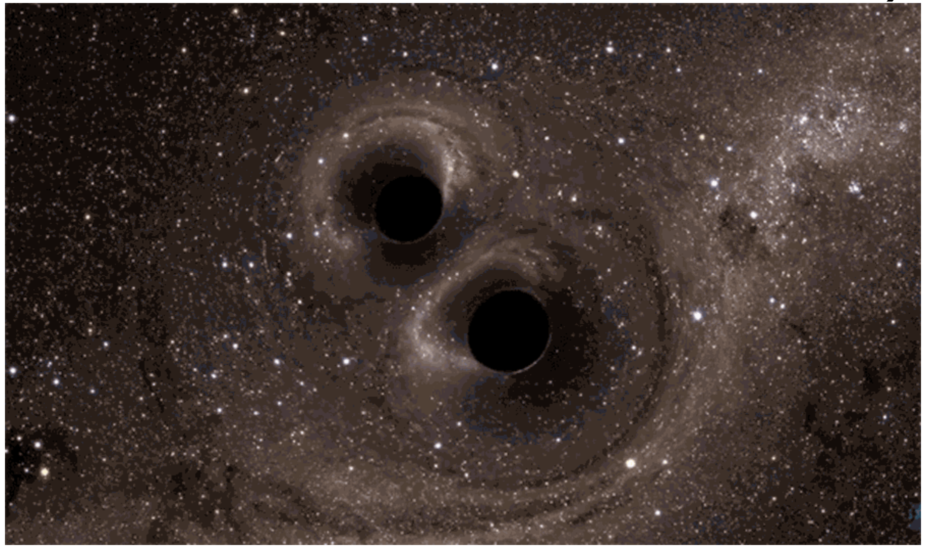
On September 14,2015. Two LIGO detectors simultaneously observed a Gravitational-Wave signal.



Credit: *LIGO/T. Pyle*

Source

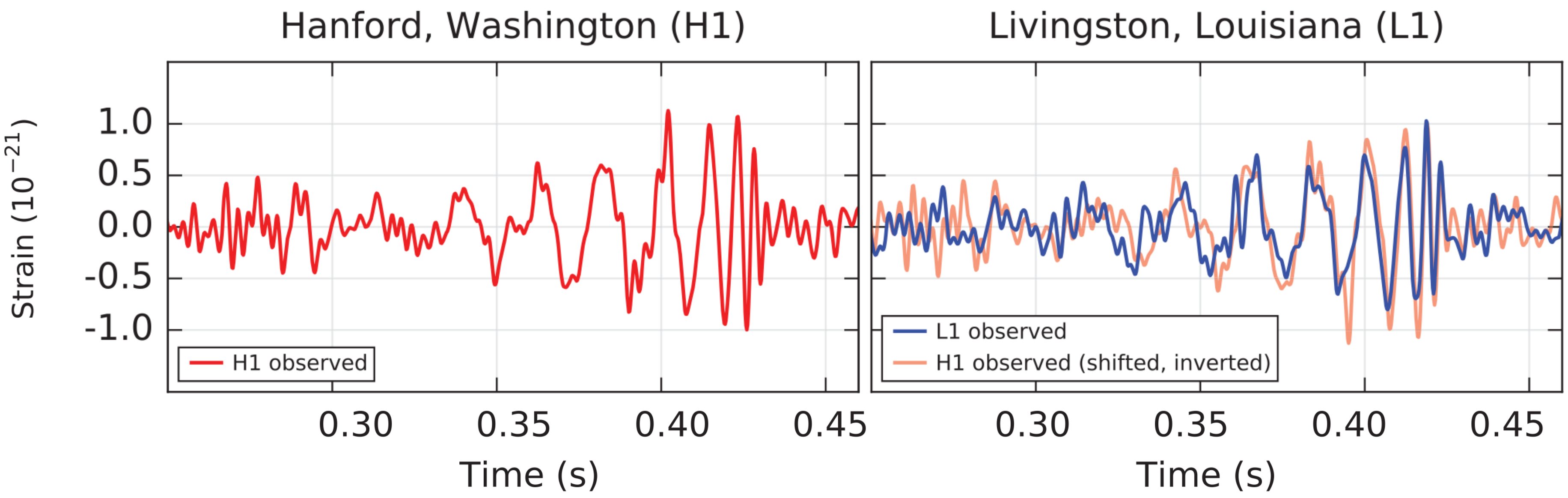
- Source: Binary Black hole Merge
- Distance:～410 Mpc (1pc=3.26ly)
- Initial Black hole mass: ～$36M_\odot$ and ～$29M_\odot$
- Final black hole mass :～$62M_\odot$ (～$3M_\odot c^2$ radiated at GW)



Credits: *SXS Lensing*

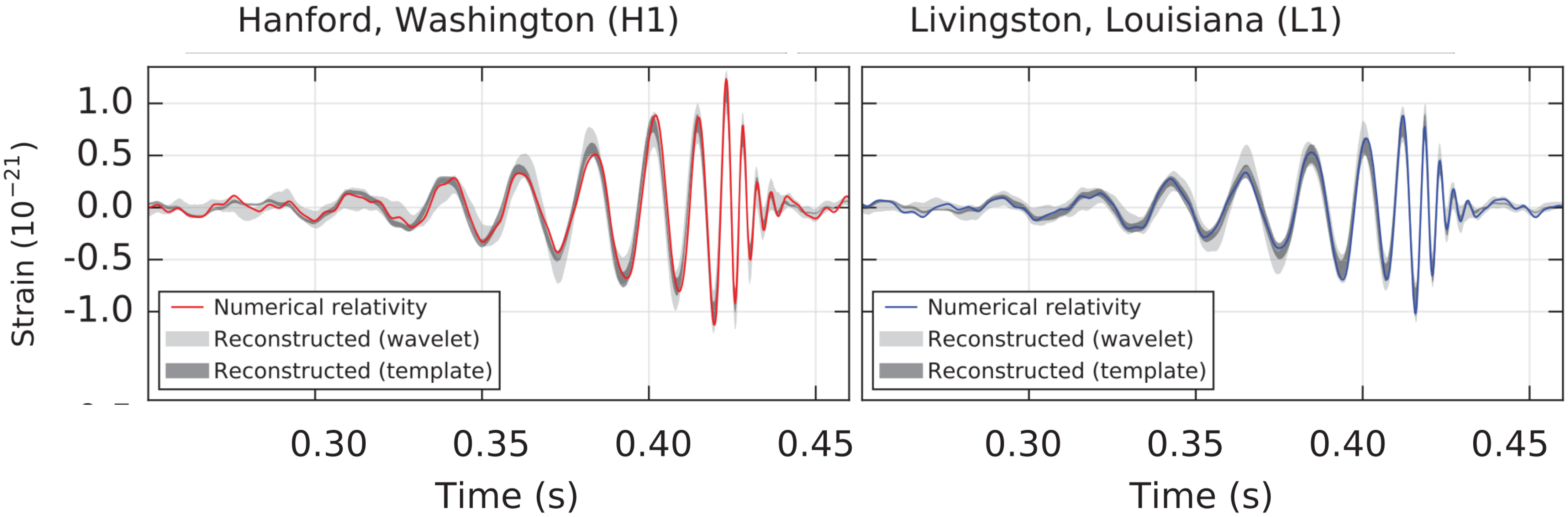
Observation and Data analysis

Detected signal by two LIGO



Credit: *PRL 116,061102 (2016)*

- Raw signal combines with noise
- Strain of the signal only had $\sim 10^{-21}$

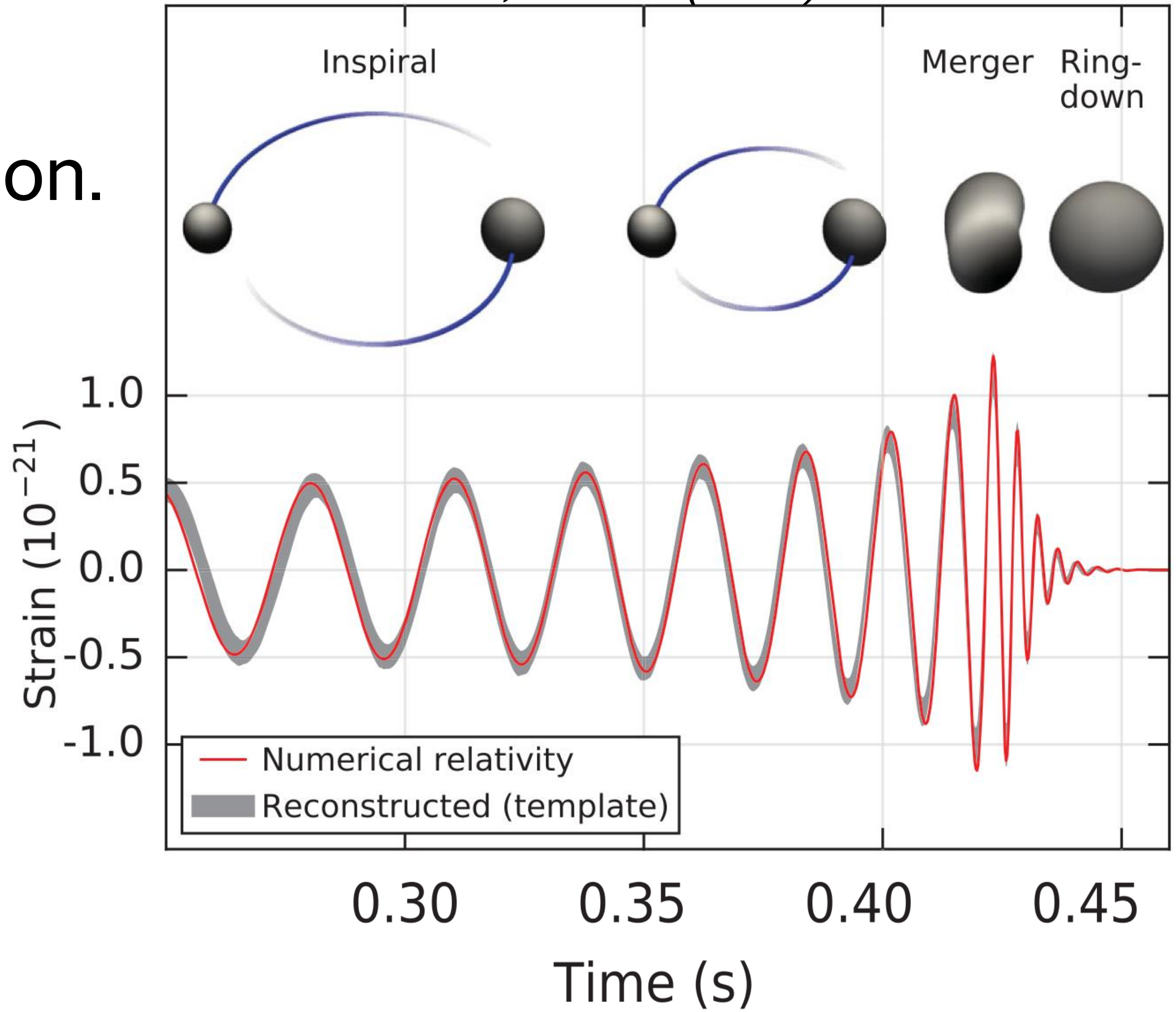


Credit: *PRL 116,061102 (2016)*

The above graphs show numerical relativity stimulation.

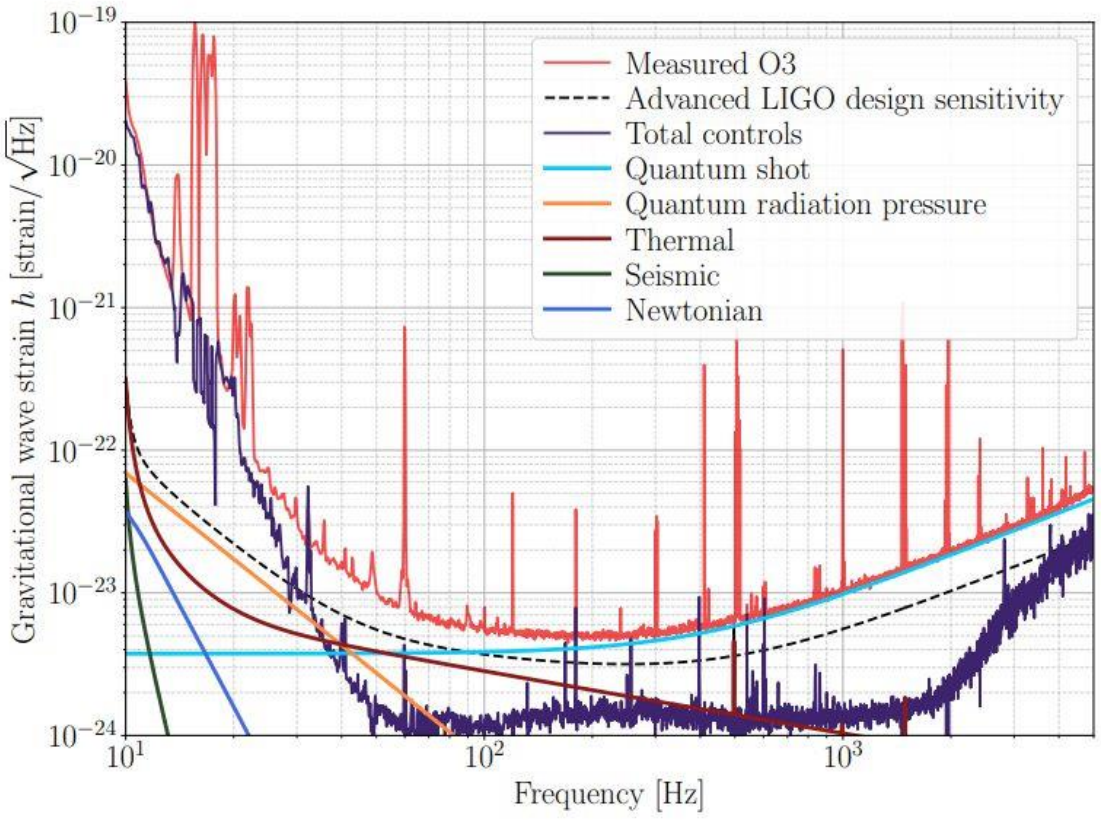
- Each part of the signal represents a different situation.
  - In 0.2 sec, the signal increases in frequency and amplitude in 8 cycles from 35 to 150 Hz
  - To reach 75 Hz, two blackholes only ~350 km apart.

Credit: *PRL 116,061102 (2016)*



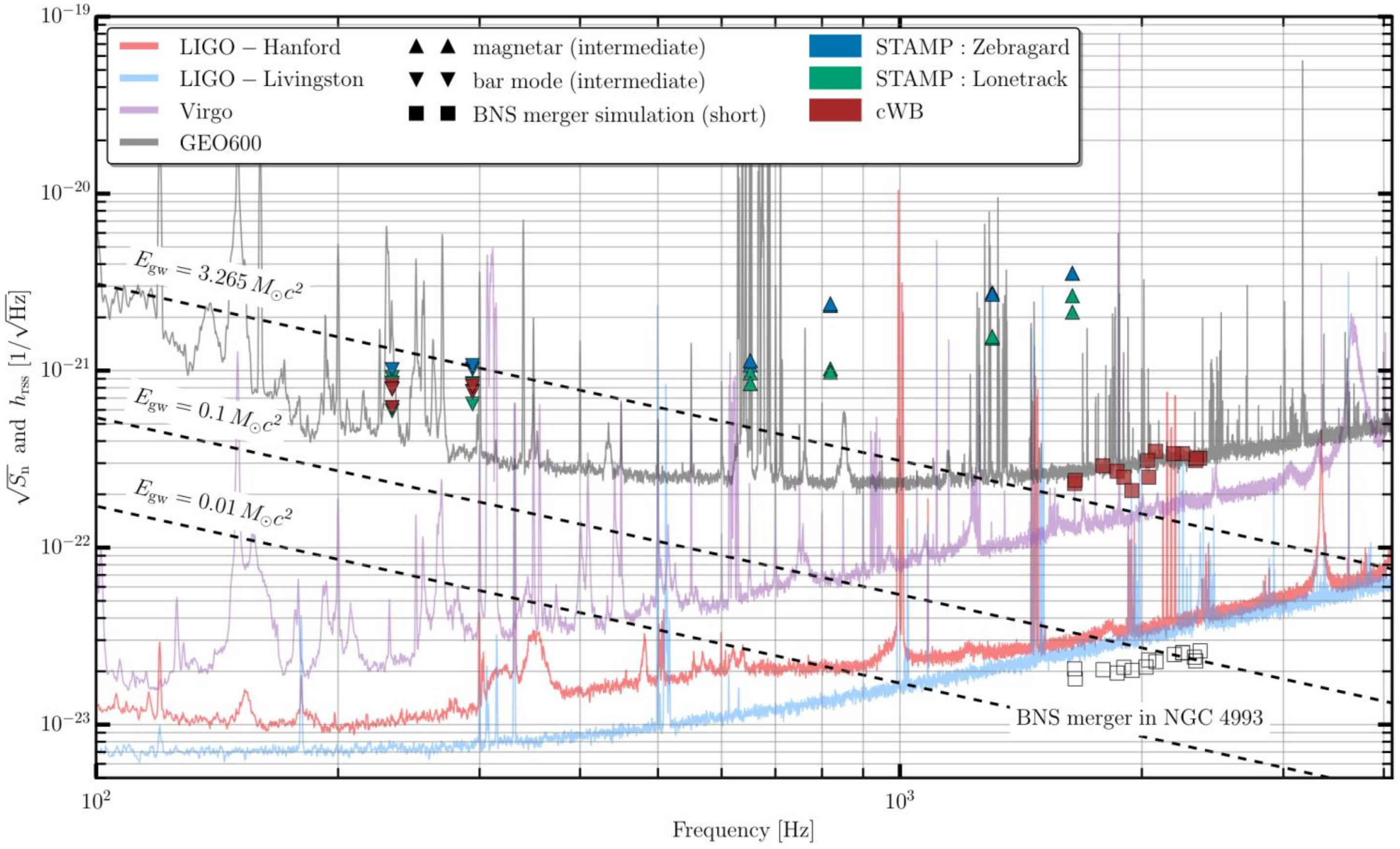
What LIGO can detect?

- Seismic Noise (<20Hz)
  - Vibration from the Earth
- Shot Noise (>450Hz)
  - Discrete properties of photon
- Target GW Signal for Binary black hole (~100Hz)

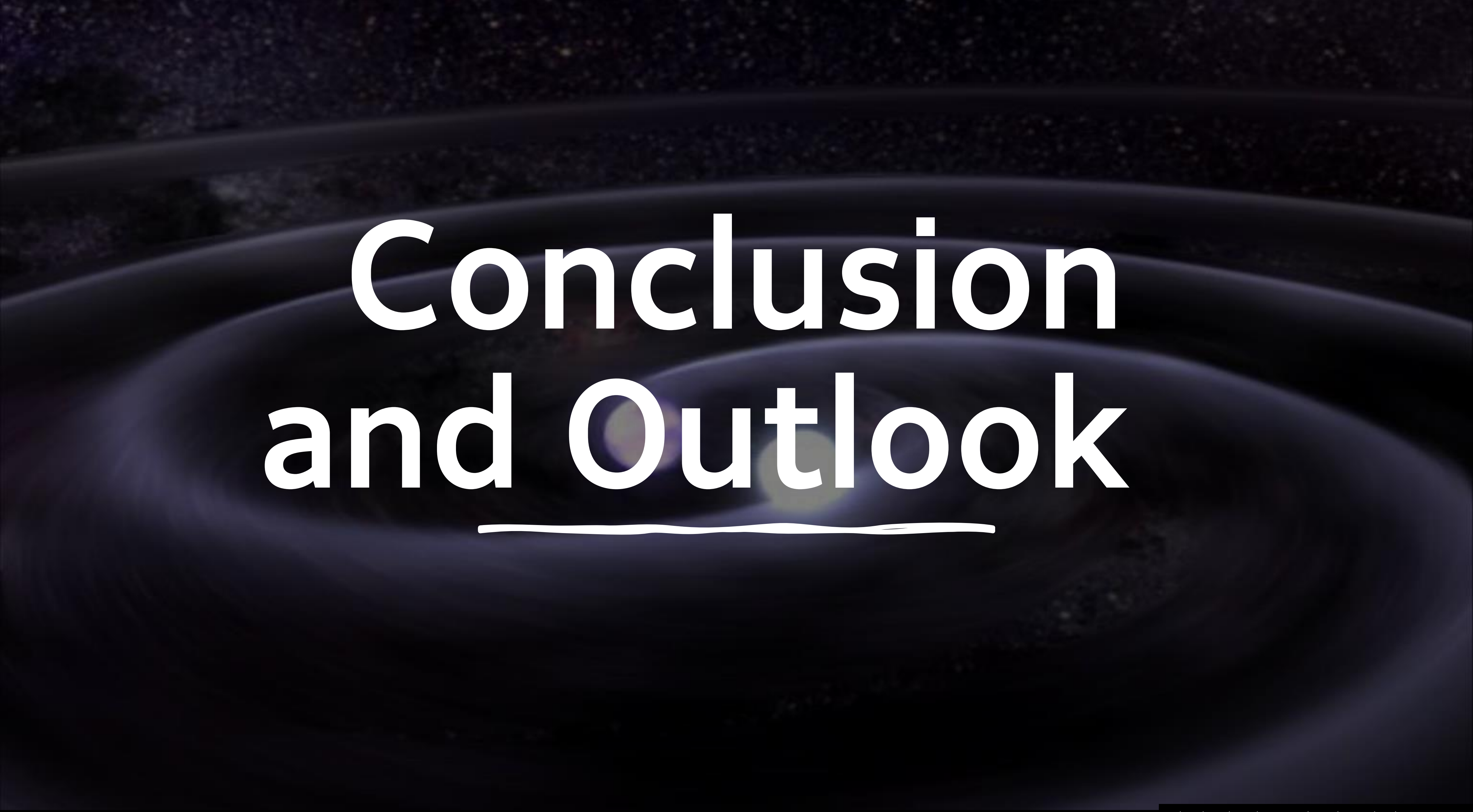


What LIGO cannot detect?

- Binary Neutron Star Merger and Post-Merger
- 1k-4k Hz



Conclusion and Outlook



Conclusion

- GW150914 event is the first direct observation of GW

  The first observation of a binary blackhole.

  The existence of GW has been proved for over 50 years

  The detected waveforms matches the prediction.

- Gravitational Wave detection has completed the theory of general Relativity.

Outlook

- Detection of GW has given us a new sight for observing the universe.

Other than relying EM radiation for detection only.

- More events such as neutron stars' inspiral could be observed.